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Dynamics of a projectile penetrating in granular systems

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The impact of a sphere with velocity u_0 on a fine, loose granular system under the acceleration due to gravity has been studied by fast video photography. The behavior of the granular bed is found to be similar to a fluid during initial impact, followed by a cavity drag during projectile penetration. From the trajectory of the projectile it is found that the drag on the projectile can be well described by adding a bulk frictional force f to the hydrostatic force κz where κ is a constant and z denotes the penetration depth. Both κ and f are u_0 dependent. This form of the drag force suggests that fluidlike viscous dissipations in the bed can be neglected in these three-dimensional (3D) experiments. However, due to the imposed boundary this hydrodynamic term of the drag force is found to be not negligible in quasi-2D granular beds.

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An interesting property of granular materials is that they can sustain external stresses similar to a solid and flow similar to a fluid [1–3]. This solid to fluid transition occurs when energies from mechanical stress are being transferred to the kinetic energy of the granular particles. Recent studies [4–8] have demonstrated such a transition with a table top experiment by dropping a projectile on top of a granular bed. If conditions are correct, the projectile will penetrate deep into the granular bed and a narrow stream of granular materials can be ejected from the bed; similar to the formation of Worthington jets [9] in a fluid. However, very little is known about the physical properties of the bed during the impact. For example, even fundamental quantities such as the drag forces (F_D) acting on the projectile during granular penetration are not well understood yet [10].

In the case of a fluid, F_D can be understood from the dynamical properties of the fluid which are well known. In fluids, the force impulse produced during the impact phase is due to the acceleration of the virtual mass of fluid associated with the projectile at the water surface [11]. After the initial impact, during the penetration and before the cavity closure, the hydrostatic pressure gradient and the gravitational forces have to be taken into consideration in the drag force in addition to the hydrodynamical pressure. The drag force can increase significantly due to this cavity drag [11]. These fundamental phenomena are characterized by three parameters, virtual mass, drag coefficient, and cavity closure time.

For granular materials, properties of a granular bed are strongly related to the distribution of force chains [12–14] in the bed. Recent study of Geng and Behringer [15] has shown that the drag force experienced by a slowly moving object in a two-dimensional granular system is dominated by force chain structures in the bulk of the system. Presumably dynamics of these chains will determine the forces acting on the projectile during penetrations. However, it is not clear whether this dynamics will give rise to fluidlike phenomena such as viscous damping for the granular penetrations. There are investigations carried out by Ciamarra *et al.* [16] and Lohse *et al.* [5,17] to probe the dynamical properties of a granular bed by impact experiments in 2D and 3D. A remarkable discovery of Ref. [17] is that for projectiles with zero impact velocity ($u_0=0$), F_D can be described simply by the term κz, where the parameter κ characterizes the force of bed at a depth of z from the surface; similar to the case of a static bed. With this form of the drag force, it seems that the fluidlike properties of the bed can be neglected.

However, in Ref. [16], the projectile is found to experience a u_0-dependent uniform deceleration during penetration with nonzero impact velocities u_0; suggesting that fluidlike properties might be important. Intuitively, the drag force in Ref. [17] with zero impact velocities will not be sensitive to the fluidlike properties of the bed, while velocity-dependent properties of the bed might be dominant in Ref. [16] with nonzero impact velocities. These penetration experiments seem to represent the two limiting cases of the same phenomenon. However, there might also be fundamental differences in these two experiments due to the different conditions of the beds; namely, a three-dimensional (3D) bed in Ref. [17] and a quasi-2D bed in Ref. [16].

In this Brief Report, results of impact experiments designed to measure the drag forces acting on the projectiles by varying systematically the initial velocity u_0 in both 3D and quasi-2D granular beds are reported. The behavior of the bed is found to be similar to a fluid during impact and become different from the behavior of the fluid during penetration. A constant drag force term f is found to exist in granular systems during the penetration. In particular, the drag force is in form of $F_D=\kappa z+f$, with both κ and f being u_0 dependent. This form of the drag force suggests that fluidlike viscous dissipations in the bed can be neglected in these 3D experiments, while in 2D experiments viscous dissipation is found to be not negligible as indicated by the measured $u(t)$ curves.

The granular beds in the 3D experiments are prepared by

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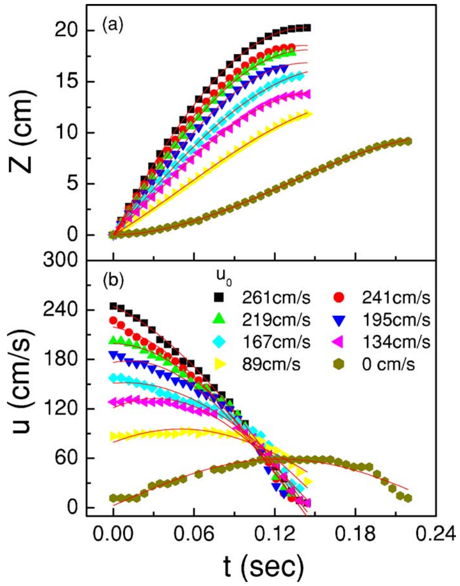

FIG. 1. (Color online) The time dependence of the penetration distance $z(t)$ (a) and velocity $u(t)$ (b) of a copper sphere (diameter $\phi=2.0$ cm and mass $m=12.3$ g) at various impact velocities u_0. The solid lines are curves fitted by $z(t)=-(mg'/\kappa)\cos\sqrt{\kappa/m}t+(u_0'/\sqrt{\kappa/m})\sin\sqrt{\kappa/m}t+(mg'/k)$ and $u(t)=dz/dt$.

filling a cylindrical container (diameter=19.6 cm) to a height of 25 cm with hollow cenospheres (diameter $\approx 74-100$ μm) of density ~ 0.693 g/cm^3. The beds are always prepared with the following procedure for homogeneity. Particles are first poured into the bed with a sieve (mesh size=0.4 mm) buried at the bottom. The sieve is then retrieved by slowly pulling it up from the bottom. The volume fraction of the bed produced by this procedure consistently gives a value of about 0.54. Two hollow copper spheres of diameter 2 cm with masses 8.7 and 12.3 g each and seven hollow copper spheres of diameter 1.5 cm filled with different amount of tin solder to make total masses of 4.9, 5.9, 6.9, 7.9, 8.9, 9.9, and 10.9 g, respectively, are used as projectiles. A metal filament (diameter=0.5 mm, length=26 cm, mass =0.4 g) is attached to the projectile to work as an indicator for the position of the projectile during penetrations when the projectile is not visible. The projectile was hung by the end of the filament with a string at various height from the surface of the bed. The impact experiment is initiated by burning the string. Motions of the projectile is then recorded by a 1000 frames/sec video camera. In all the figures shown below the errors of the data are smaller than the size of the symbols, but there are 5–18 % variations among experiments because of the sensitivities of the granular beds to their initial conditions.

Figure 1(a) shows the time dependence of the penetration depth (z) of a copper sphere (diameter $d=2.0$ cm and mass 12.3 g) of various impact velocities (u_0) penetrating into the granular bed measured by the video camera. Note that u_0 are calculated from the drop height of the spheres and confirmed by measurement from video images just before the entry into the bed. Granular jets can be observed in these experiments. If these data are fitted to the form of constant deceleration (not shown) proposed in Ref. [16], we find that the constant deceleration fitting is only valid for $u_0 > 134$ cm/sec and a limited range of time. Time dependence of the velocities $[u(t)]$ of the projectiles can be determined from $z(t)$ in Fig. 1(a) and are shown in Fig. 1(b). It can be seen that $u(t)$ is not a linear function of t. In fact, except for small u_0, the general shapes of the curves are convex; $d^2u/dt^2 < 0$. Since du/dt is proportional to the net damping force acting on the projectile, this last finding suggests that the damping force increases with time and therefore with depth z.

When $u_0 < 167$ cm/sec, as shown in Fig. 1(b), the projectiles are accelerated first before there are decelerations, a shape similar to Ref. [17]. In Ref. [17], the u_0 is always zero and the form of the equation of motion of the projectile proposed and verified is $(M+m)(d^2z/dt^2)=-\kappa z+mg$, where M is the mass of the granular materials moving with the projectile and κ is a force constant related to the configuration of the granular bed. This last form of equation of motion fits the data of Ref. [17] well with $M=0$. However, in fitting our system a modified form $m(d^2z/dt^2)=-\kappa z+mg-f$ with u_0' (the initial velocity), κ and f being the fitting parameters is found to fit well our data with zero and nonzero impact velocities. In Ref. [17] the volume fraction of their sand is much smaller and they have only considered impact with $u_0=0$, the f term may be too small to show any effect. The physical meaning of this form will be discussed below.

The solid lines $z(t)$ and $u(t)$ in Figs. 1(a) and 1(b) are the fits of our data to the time integrals of the above modified form. It can be seen that this form fits our data very well for $z(t)$ and to a less extent for $u(t)$. These results show that the fitting of data to $u(t)$ is more sensitive than that of $z(t)$. Figure 2 shows the u_0 dependence of the three fitting parameters of spheres with same diameter 1.5 cm. Note that u_0 is the velocity of the projectile just before impact and u_0' is the fitted value of the velocity of the projectile just after impact. Therefore, $m(u_0-u_0')$ is the change in momentum of the projectile due to impact. In Fig. 2, $u_0' \approx \alpha u_0$ with $\alpha=0.80$, 0.82, 0.85, 0.87, 0.87, 0.86, and 0.90 for the copper spheres from lighter to heavier ones, respectively. If the granular bed is considered as fluid, the mass M introduced above can be estimated from Fig. 2 by using the result of projectiles impact on water surfaces [18]. If ρ_m is the mean density of the bed, for the impact, we have $mu_0=(M+m)u_0'$, $u_0'/u_0 =m/(M+m)$ and $M=2\pi\rho_m r^3/3$ [18]. The linear relation between u_0 and u_0' in Fig. 1(a) shows that the granular bed is very similar to a fluid during impact. Furthermore, if we put in the numerical value of $\rho_m=0.37$ g/cm^3, we get the value u_0'/u_0 as 0.94, 0.95, 0.96, 0.96, 0.97, 0.97, and 0.97 for the copper spheres from lighter to heavier ones, respectively. These theoretical values are 8–18 % higher than those obtained from Fig. 2 above with the largest deviation for glass sphere. Presumably, there are still some fundamental differences between fluids and a fluidized granular beds which are sensitive to the surface properties of the projectiles.

The fittings in Figs. 1(a) and 1(b) are obtained with a u_0 dependent g'. The fitted values of g' in Fig. 1 are smaller than g with g' closest to g when $u_0=0$. That means that there is an additional u_0 dependent constant force acting on the projectile. Note that the term mg' can be written as mg'

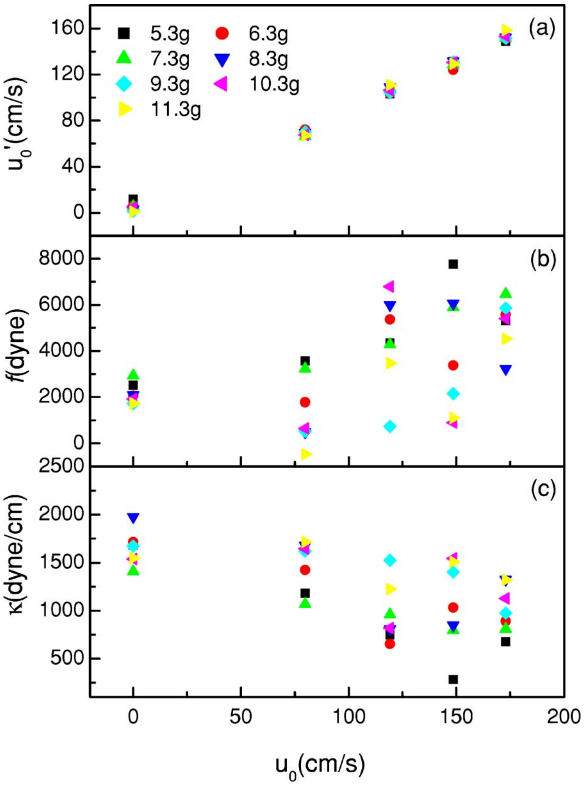

FIG. 2. (Color online) The u_0 dependence of the fitting parameters u_0', f, and κ of projectiles with the same diameter 1.5 cm and different masses. The mass shown in the figure includes the mass of the projectile and the attached filament.

$\equiv m(g-f/m)$ for some f. The physical meaning of the f term is that there is a stress in the bed which opposes the motion of the projectile during penetration. Presumably, f originates from the collisions between the projectile and the granular materials during the penetration and therefore should increase with the velocity of the projectile. In fact, one observes in the experiments that f increases with u_0 as shown in Fig. 2(b). In a fluid, this term is the dynamical pressure: ρu^2. However, in granular beds a minimum "yield stress" f is needed for the projectile to overcome in order to fluidize the bed. It is a more frictionlike constant term. In Ref. [17], $f \approx 0$. Presumably, the loosely packed bed of their quick sands reduces the significance of this term.

If the granular bed can be considered as a fluid, the κ term in Ref. [17] is just the hydrostatic pressure acting on the bottom part of the projectile because there is a cavity on top of the projectile during penetration [18]. In our case, κ can be expressed as $\kappa \approx \rho_m g \pi r^2$. For $r \approx 1$ cm and $\rho_m = 0.37$ g/cm^3, we have $\kappa \approx 10^3$ dyne/cm$=1$ N/m. This theoretical value of κ is of the same order of magnitude of the fitted values of κ from our experiments as shown in Fig. 2(c). Same estimate can be done to compare this theoretical κ with the fitted value of κ in Ref. [17]. They are also found to be in the same order of magnitude. With this simple model, κ is u_0 independent. Although the u_0 dependence of κ cannot be deduced because of the scattering of data, it is clear that κ seems to be a decreasing function of u_0 with κ being largest when $u_0=0$. Intuitively, the degree of fluidization of the bed increases with u_0 and a bed with higher degree of fluidization will give a smaller κ. Due to the structure of the force chains in the granular bed is sensitive to the initial preparation of the bed, a large fluctuation of f and κ are expected.

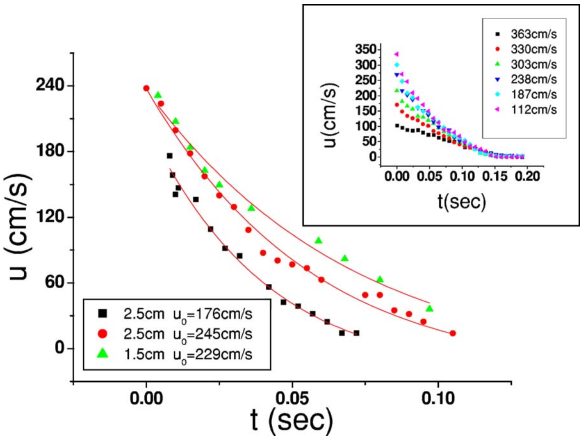

FIG. 3. (Color online) Time dependence of the velocity $u(t)$ of projectiles in a quasi-2D container. Inset is $u(t)$ in Ref. [16].

Although it seems that the effects of the viscous damping are not obvious in our 3D experiments, their effects can be significant when the projectile is close to the wall. To test the effects of the force chains on the impact experiments, a quasi-2D experiment is carried out with the same granular materials used in the 3D experiments but in a rectangular bed of size 24 cm (height) $\times$ 25 cm (width) $\times$ 2 cm (thickness). The idea is to create a situation in which the projectile is always close to the wall. The drag force is measured in a slow pushing experiment. It is found that the drag force measured for this 2D bed is in fact much larger than that measured in 3D beds.

Impact experiments are also performed in this bed with the spherical projectiles replaced by 1.9-cm-thick copper cylinders of diameter 1.5 and 2.5 cm and of masses 28.5 and 78.7 g, respectively. Results of the time dependence of $u(t)$ with this setup is shown in Fig. 3. A remarkable feature is that the general shapes of the curves are concave, i.e., $d^2u/dt^2>0$; the same as found in Ref. [16] (see inset of Fig. 3), but is different from those in 3D. The difference in the shape of $u(t)$ curves in quasi-2D and -3D indicates that the mechanism of penetration in the 2D and 3D cases are quite different. In the 3D experiments, because of the depth dependent force κz, the deceleration is largest at the end of the penetration. However, in the 2D cases, the deceleration of the projectile decreases with time; suggesting that it is a relaxation process related to the velocity of the projectile. In fact, one can fit the $u(t)$ curves in 2D to an exponential form shown as solid lines in the figure. In fluids, this exponential behavior originates from the viscous damping of the fluids. Since the same kind of granular materials are used in both our 2D and 3D experiments, the difference in the form of $u(t)$ observed in the 2D and 3D cases is obviously not due to the difference in the shapes of the grains. Also, one can rule out the effect of the difference in the packing of 2D and 3D systems because packing in our quasi-2D system is actually 3D-like while the packing in Ref. [16] is 2D-like but they both give similar form of $u(t)$. It seems that the shape of $u(t)$ is governed by the relative importance of dissipations (viscosity). In 2D case the granules being pushed away by the projectile can move only laterally, while in 3D case the particles have one more dimension of freedom to move, which may be the reason that causes the dissipative effect more significant in 2D systems.

In conclusion it is found in this work that the granular bed is behaving similar to a fluid in the aspects that during the impact a virtual mass can be estimated using the ratio u_0'/u_0 and during the penetration the drag force can be described by κz due to the cavitation, and that both u_0'/u_0 and κ can be estimated from a simple fluid model. The difference between granular systems and fluids is that in granular bed there exists a u_0 dependent frictionlike drag term f during the projectile penetration. The drag force f can be understood as a term equivalent to "yield stress" for the projectile to accelerate the granules to move laterally during penetration. It is also found that due to the imposed boundary the dynamics of a projectile penetrating in 2D and 3D systems seem to be in different forms. In 3D systems the hydrostatic pressure is dominant during the penetration, while in 2D experiments the viscous damping seems to be a dominant term. These findings of projectile impact and penetration in granular systems and the proposed phenomenological model deserve further theoretical investigations for a more basic understanding of the impact process.

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[1] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, Rev. Mod. Phys. **68**, 1259 (1996).

[2] L. P. Kadanoff, Rev. Mod. Phys. **71**, 435 (1999).

[3] P. G. de Gennes, Rev. Mod. Phys. **71**, S374 (1999).

[4] S. T. Thoroddsen and Amy Q. Shen, Phys. Fluids **13**, 4 (2001).

[5] D. Lohse, R. Bergmann, R. Mikkelsen, C. Zeilstra, D. van der Meer, M. Versluis, Ko van der Weele, M. van der Hoef, and H. Kuipers, Phys. Rev. Lett. **93**, 198003 (2004).

[6] J. S. Uehara, M. A. Ambroso, R. P. Ojha, and D. J. Durian, Phys. Rev. Lett. **90**, 194301 (2003).

[7] A. M. Walsh, K. E. Holloway, P. Habdas, and J. R. de Bruyn, Phys. Rev. Lett. **91**, 104301 (2003).

[8] J. C. Amato and R. E. Williams, Am. J. Phys. **66**, 141 (1998).

[9] A. M. Worthington, *A Study of Splashes* (Longmans, Green and Co., London, 1908).

[10] M. B. Stone, Nature (London) **427**, 503 (2004).

[11] J. W. Glasheen and T. A. McMahon, Phys. Fluids **8**, 2078 (1996).

[12] D. Howell, R. P. Behringer, and C. Veje, Phys. Rev. Lett. **82**, 5241 (1999).

[13] D. M. Mueth, H. M. Jaeger, and S. R. Nagel, Phys. Rev. E **57**, 3164 (1998).

[14] H. A. Makse, D. L. Johnson, and L. M. Schwartz, Phys. Rev. Lett. **84**, 4160 (2000).

[15] J. Geng and R. P. Behringer, Phys. Rev. E **71**, 011302 (2005).

[16] M. P. Ciamarra, A. H. Lara, A. T. Lee, D. I. Goldman, I. Vishik, and H. L. Swinney, Phys. Rev. Lett. **92**, 194301 (2004).

[17] D. Lohse, R. Rauhe, R. Bergmann, and D. van der Meer, Nature (London) **432**, 689 (2004).

[18] G. Richardson, Proc. Phys. Soc. London **61**, 352 (1948).